gages were operative in all of the shots. It can be seen from the table that pressures of considerable significance (several hundred psi) occur a foot away from the cavity center.

### References

[1]Lundstrom, Eric A. *Fluid Dynamic Analysis of Hydraulic Ram,* NWC Technical Publication 5227, July 1971, Naval Weapons Center, China Lake, Calif.

[2]Torvik, P. J. and Clark, J. W. "The Reduction of Impact Induced Pressures in Fuel Tanks," *43rd Shock and Vibration Bulletin,* Pt. 2, June 1973, pp. 203–208.

[3]Ball, R. E., Power, H. L., and Fuhs, A. E. "Fuel Tank Wall Response to Hydraulic Ram during the Shock Phase," *Journal of Aircraft, Vol. 10,* No. 9, Sept. 1973, pp. 571–572.

[4]Cole, R. H. *Underwater Explosions,* Princeton University Press, Princeton, N.J., 1948, pp. 275, 276, 364.

[5]Hoerner, S. F. *Fluid Dynamic Drag,* Hoerner, Midland Park, N.J., 1958.

# Computation of Transonic Flow about Lifting Wing-Cylinder Combinations

E. B. Klunker* and P. A. Newman*
*NASA Langley Research Center, Hampton, Va.*

SUCCESSFUL development of relaxation algorithms for the computation of inviscid potential flow about two-dimensional airfoils and bodies of revolution at transonic speeds[1-8] has led to adaptation of the technique to more complex configurations. Three-dimensional wing calculations based upon a small disturbance potential equation have been presented in Refs. 9–13 and calculations for a yawed wing with the full potential equation are described in Ref. 14. To date, the only published results for a wing-body configuration are for a nonlifting wing with a cylindrical body.[12] Some results are presented here for a lifting rectangular wing centrally located on a circular-cylindrical body. This simple configuration has been utilized in order to assess the merits of a mapping technique for wing-body configurations.

Two general approaches can be followed in the formulation of these problems: 1) employ Cartesian (or other simple) coordinates which preserve the utmost simplicity in the governing differential equation but which generally necessitate interpolation or extrapolation to incorporate some of the boundary conditions, or 2) utilize coordinate transformations which simplify specification of the surface boundary conditions but which generally increase the complexity of the differential equation. The first technique has been used in Ref. 15 for a two-dimensional time-dependent computation while the second has been widely employed in fluid mechanics and other fields. A combination of the two approaches will most likely be required for all but the simplest configurations. The procedure employed here makes use of a coordinate transformation to simplify specification of the surface boundary condition in the computation of the flow about a rectangular wing symmetrically located on a circular-cylindrical body. The method can be extended to incorporate wing sweep, finite length body of noncircular cross section, and arbitrary wing placement; however, these extensions involve a considerable increase in complexity of the problem.

Received January 18, 1974.
Index categories:

* Aerospace Technologists, High-Speed Aircraft Division.

### Method

One of the simplest wing-body configurations which can be reduced to a simple form by a coordinate transformation is that of a cylindrical body with the wing plane passing through the body axis. A Joukowski-type transformation in planes normal to the body axis

$$\sigma = \sigma_1 + c^2/\sigma_1 \quad x = x_1 \quad \sigma_1 = y_1 + iz_1 \quad \sigma = y + iz \tag{1}$$

where $c$ is a constant, maps the circular cross section $y_c^2 + z_c^2 = c^2$ ($y = y_c$, $z = z_c$ on $c$) and the plane $z = 0$ for $|y| \geq c$ onto the plane $z_1 = 0$. If the body radius does not differ appreciably from $c$, then the boundary condition of zero normal velocity both on the wing and on the body can be enforced on the plane $z_1 = 0$. Moreover, the boundary condition on the trailing vortex sheet, which is assumed to lie in the plane of the wing, is similarly satisfied on the plane $z_1 = 0$. The condition that $c$ is a constant preserves the orthogonality of the coordinates but excludes consideration of bodies of finite length. The body radius $r_o$ can depend both on $x$ and the polar angle $\Theta$ = arctan $(z_o/y_o($; however, one-to-one correspondence between the coordinate systems $x,y,z$ and $x_1,y_1,z_1$ requires that $r_o \geq c$. Thus the boundary-value problem for this simple wing-body configuration in the $x_1,y_1,z_1$ coordinates is similar in most respects to that of a wing alone in Cartesian coordinates.

The equation for the pertubation velocity potential used here is

$$[\beta^2 - (\gamma + 1)M_\infty{}^2\phi_x(1 + \phi_x/2)]\phi_{xx} + \phi_{yy} + \phi_{zz} = 0 \tag{2}$$

where $M_\infty$ is the stream Mach number, $\beta^2 = 1 - M_\infty{}^2$, and $\gamma$ is the ratio of specific heats. Equation (2) differs from the familiar form of the small disturbance transonic flow equation by inclusion of the term $\phi_x{}^2\phi_{xx}$ which has been retained to better approximate the critical speed where the equation changes type from elliptic to hyperbolic.

The flow tangency condition on the wing is approximated by

$$\phi_z(x, y, 0) = f_x - \alpha \tag{3}$$

for $x,y$ on the wing planform where $z = f(x,y)$ is the equation of the wing surface and $\alpha$ is the angle of incidence. It may be noted here that taking the surface boundary condition as $\phi_z(x,y,0) = [1 + \phi_x(x,y,0)]f_x - \alpha$ generally leads to results where the full expansion around the blunt leading edge of an airfoil is not achieved. The flow tangency condition for a body of revolution is approximated by

$$y_c\phi_y(x, y_c, z_c) + z_c[\alpha + \phi_z(x, y_c, z_c)] = c(dr_0/dx) \tag{4}$$

Additional terms involving $(dr_o/d\theta)$ must be included in Eq. (4) for bodies of noncircular cross section. The boundary condition on the trailing vortex sheet, assumed to lie in the plane of the wing $z = 0$, is

$$\phi(x, y, 0^+) - \phi(x, y, 0^-) = \Gamma(y) \tag{5}$$

for $x,y$ on the sheet; the circulation $\Gamma(y)$ is determined as part of the solution.

The disturbance velocity potential must vanish far from the wing-body configuration and its trailing vortex sheet. Thus, $\phi = 0$ on the outer boundary surfaces except those far upstream and far downstream in the vicinity of the

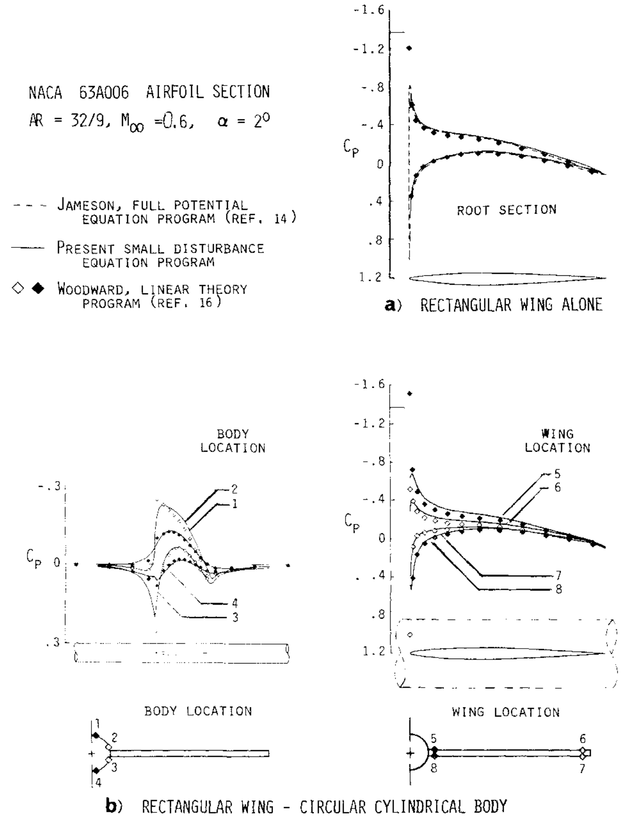


**Fig. 1 Computed surface pressure coefficients for subcritical flow.**

body and vortex sheet. At each of these limits the differential equation reduces to the two-dimensional Laplace equation which is solved subject to the surface boundary condition on the body, the jump condition on the vortex sheet, and $\phi = 0$ on the outer boundary. In the present computations, these conditions have been imposed at a finite distance upstream and downstream of the wing as well as in the spanwise direction, since the $x_1$ and $y_1$ coordinates have not been stretched. However, a coordinate stretching has been employed for the $z_1$ coordinate which maps $|z_1| \leq \infty$ into a finite domain.

Line relaxation (along lines for which $x_1$ and $y_1$ are constant) is used to solve the finite-difference approximation to Eq. (2). Three-point central differences are used for computing all velocity components as well as the second derivatives of $\phi$ at subsonic points. Jameson[7,14] has analyzed iterative solution algorithms of mixed-type finite-difference equations which arise in transonic flow and has shown that great care must be taken to insure a proper domain of dependence and iterative stability (von Neumann criterion) at places where the flow is locally supersonic. The direction in which a line-relaxation calculation proceeds through the computational grid must be considered in the stability analysis and this aspect introduces differences between two-dimensional and three-dimensional finite-difference operators for a line-relaxation algorithm. These ideas have been incorporated in the program outlined here but space limitations preclude elaboration of details. As in Ref. 10, the circulation is relaxed after all lines have been updated.

## Results

Chordwise pressure distributions computed from the present small-disturbance program for subcritical flow at $M_\infty = 0.6$ and 2° incidence are compared in Fig. 1 with results from a linear-theory program[16] and from a full potential-equation program[14] for the wing alone. The value of $C_p{}^*$ shown by the tick mark in the figures corresponds to the condition where the coefficient of $\phi_{xx}$ in Eq. (2) vanishes and consequently differs slightly from the exact sonic value. The pressure distribution shown for the wing alone (Fig. 1a) is at the root section of a rectangular wing of aspect ratio = 32/9 which has a NACA 63A006 airfoil section. All three computations are in good agreement. In Fig. 1b, pressure distributions from the present program are compared with those from the Woodward program[16] at several wing and body locations which are indicated at the bottom of the figure. The wing and flow conditions are the same as those for the wing alone but with a circular-cylindrical body of radius 1/10 semispan. Note that the $C_p$ scale for the body results in this figure differs from that used in all other figures. The agreement between these linear and relaxation calculations is good, indicating little nonlinear effects at this Mach number.

Similar computations for supercritical flow at $M = 0.9$ and 2° incidence are shown in Fig. 2. The present results for the wing alone are compared with those from the Jameson program[14] at the root section in Fig. 2a. The shock locations on both the upper and lower surfaces differ by one mesh space; however, the agreement is generally good. It should be noted that the computational grid for the present results is substantially coarser than that used in the program of Ref. 14. Surface pressures from the present relaxation calculation at several locations on both the wing and body, are shown in Fig. 2b. No other results were available for comparison.

The results indicate that useful three-dimensional computations can be obtained with a small disturbance equation. A disadvantage of these equations, however, is a sensitivity of the solution to the grid, particularly in the region of a blunt leading edge. Thus some experimentation in the choice of the coordinate lattice may be required to achieve best results. Comparisons with computations based on the full equations are valuable for this purpose. This "tuning" of the computational grid may seem incongruous in view of the fact that the assumptions of small disturbances are violated there. Nevertheless, it provides a means for obtaining acceptable results for blunt leading-edge configurations using approximate equations.

The cross-flow mapping employed to simplify specification of the boundary conditions on the body caused no ad-

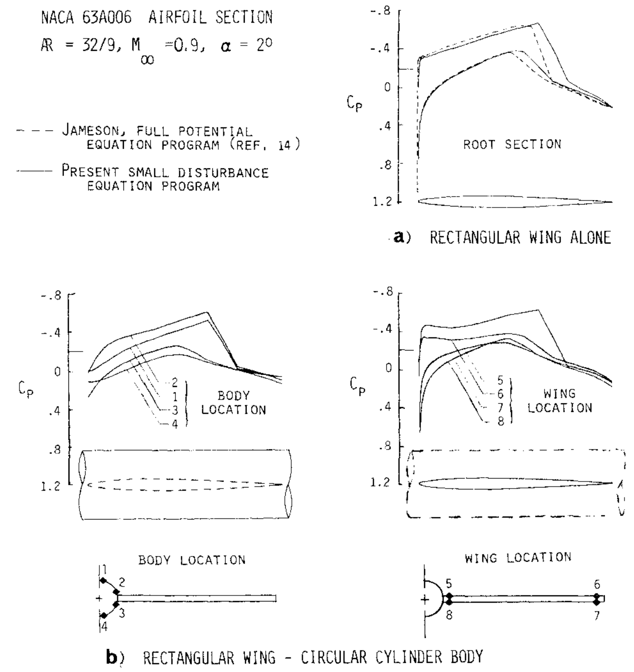


**Fig. 2 Computed surface pressure coefficients for supercritical flow.**

ditional difficulties. Adaptation to more complex configurations such as a finite-length body, general body cross section, and arbitrary placement of a swept wing on the body will entail considerably more complexity than that required for this simple configuration; however, the general approach appears to hold merit.

## References

[1]Murman, E. M. and Cole, J. D., "Calculation of Plane Steady Transonic Flows," *AIAA Journal*, Vol. 9, No. 1, Jan. 1971, pp. 114–121.

[2]Krupp, J. A. and Murman, E. M., "Computation of Transonic Flows past Lifting Airfoils and Slender Bodies," *AIAA Journal*, Vol. 10, No. 7, July 1972, pp. 880–886.

[3]Bailey, F. R., "Numerical Calculation of Transonic Flow About Slender Bodies of Revolution," TN D-6582, 1971, NASA.

[4]Steger, J. L. and Lomax, H., "Transonic Flow About Two-Dimensional Airfoils by Relaxation Procedures," *AIAA Journal*, Vol. 10, No. 1, Jan. 1972, pp. 49–54.

[5]Garabedian, P. R. and Korn, D. G., "Analysis of Transonic Airfoils," *Communications on Pure and Applied Mathematics*, Vol. 24, No. 6, Nov. 1971, pp. 841–851.

[6]Jameson, A., "Transonic Flow Calculations for Airfoils and Bodies of Revolution," Aero. Rept. 390-71-1 (undated), Grumman Aerospace Corp., Bethpage, N.Y.

[7]Jameson, A., "Iterative Solution of Transonic Flow Over Airfoils and Wings, Including Flows at Mach 1," *Communications on Pure and Applied Mathematics*, 1974 (to be published).

[8]South, J. C., Jr. and Jameson, A., "Relaxation Solutions for Inviscid Axisymmetric Transonic Flow Over Blunt or Pointed Bodies," *Proceedings of the AIAA Computational Fluid Dynamics Conference*, New York, 1973, pp. 8–17.

[9]Bailey, F. R. and Steger, J. L., "Relaxation Techniques for Three-Dimensional Transonic Flow About Wings," *AIAA Journal*, Vol. 11, No. 3, March 1973, pp. 318–325.

[10]Newman, P. A. and Klunker, E. B., "Computation of Transonic Flow About Finite Lifting Wings," *AIAA Journal*, Vol. 10, No. 7, July 1972, pp. 971–973.

[11]Ballhaus, W. F. and Bailey, F. R., "Numerical Calculation of Transonic Flow About Swept Wings," AIAA Paper 72-677, Boston, Mass., 1972.

[12]Bailey, F. R. and Ballhaus, W. F., "Relaxation Methods for Transonic Flow About Wing-Cylinder Combinations and Lifting Swept Wings," *Proceedings of The Third International Conference on Numerical Methods in Fluid Mechanics*, Springer-Verlag, New York, 1973, pp. 2–9.

[13]Lomax, H., Bailey, F. R., and Ballhaus, W. F., "On The Numerical Simulation of Three-Dimensional Transonic Flow With Application to the C-141 Wing," TN D-6933, 1973, NASA.

[14]Jameson, A., "Numerical Calculation of the Three Dimensional Transonic Flow Over a Yawed Wing," *Proceedings of the AIAA Computational Fluid Dynamics Conference*, New York, 1973, pp. 18–26.

[15]Magnus, R. and Yoshihara, H., "Inviscid Transonic Flow Over Airfoils," *AIAA Journal*, Vol. 8, No. 12, Dec. 1970, pp. 2157–2162.

[16]Woodward, F. A., "An Improved Method for the Aerodynamic Analysis of Wing-Body-Tail Configurations in Subsonic and Supersonic Flow," CR-2228, 1973, NASA.